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Nonlocality and impurity effects on the magnetic penetration depth in d-wave superconductors

M A Shahzamanian and H Yavary

Department of Physics, University of Isfahan, Isfahan 81714, Iran

E-mail: hs_yavary@yahoo.com

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Abstract
The temperature dependence of the penetration depth in the presence of nonlocality and impurity in d-wave superconductors is calculated. It is shown that the penetration depth is proportional to the relaxation time at low temperatures, where the nonlocal effects are in fact completely masked by impurities.

1. Introduction

Measurements of the electromagnetic penetration depth λ at low temperatures are beginning to yield a consistent picture of the pairing state of the high-temperature superconductors (HTSC). Until recently it had been thought that the most credible data exhibited an exponential temperature dependence at low temperatures, but a reanalysis of the data of Fiory *et al* [1] clearly showed that the deviation $\Delta\lambda$ from the zero-temperature value $\lambda(0)$ was quadratic in temperature.

In conventional s-wave superconductors, the deviation $\Delta\lambda(T)$ exhibits activated behaviour, i.e. $\Delta\lambda(T) \propto \exp(\frac{-\Delta}{T})$ (throughout this paper we use units in which $k_B = \hbar = 1$), reflecting the existence of the isotropic BCS energy gap Δ at the Fermi surface [2]. In contrast, in a pure d-wave superconductor, or any other unconventional superconductor with nodes in the gap, the London (local) penetration depth varies linearly with the temperature, i.e. $\Delta\lambda(T) \propto T$ [2].

However, below a certain sample temperature T^*, the linear T-dependence of the penetration depth in HTSC crosses over to a higher-power law, most probably a T^2-law. In the d-wave scenario of HTSC, the origin of the $\Delta\lambda(T) \propto T^2$ dependence has been explained by the presence of nonmagnetic impurities, which scatter in the unitary limit [3]. In this strong-scattering limit a small amount of impurity can induce a finite residual density of states at the Fermi level, which is sufficient to change the temperature dependence of the penetration depth from T to T^2, without significantly lowering the transition temperature.

Besides impurities, at very low temperatures nonlocality may play an important role in the T^2-dependence of $\Delta\lambda(T)$. Thus nonlocality represents a second mechanism that leads to a T^2-dependence of the penetration depth sufficiently close to $T = 0$ K.

If the electron impurity scattering is treated in the Born approximation, the quasi-particle relaxation time at low temperature in superconducting states with nodes of the gap on the Fermi surface varies as the inverse power of the temperature T. This behaviour of the relaxation time was shown to give rise to transport coefficients that are in qualitative disagreement with experiments [4]. Schmitt *et al* [5] considered impurity scattering in the unitarity limit and showed that close to resonance and at low energies, τ is constant, corresponding to $\Delta\lambda(T) \propto T^2$, which is in agreement with experiments [6, 10, 11].

In this paper we calculate $\Delta\lambda(T)$ in the presence of nonlocality and impurity. We show that at very low temperatures these effects play an important role in the electromagnetic response of a d-wave superconductor, leading to $\Delta\lambda(T) \propto T^2$, and in the presence of nonlocality and impurity the nonlocal effects become masked by the presence of impurities. In the limit $\tau \to \infty$ our results are the same as those of Kosztin and Leggett [8].

2. Formulation of the problem

To demonstrate the effects of nonlocality and impurity in $\Delta\lambda$ we calculate the electromagnetic response tensor $Q_{\alpha\beta}(q)$, which relates the current density $\vec{J}$ to an applied vector potential $\vec{A}$. The electromagnetic response of a superconductor to an electromagnetic wave is given by [2]

$$J_\alpha = -\frac{Ne^2}{m} Q_{\alpha\beta}(q) A_\beta \tag{1}$$

where

$$Q_{\alpha\beta}(q) = \delta_{\alpha\beta} + \frac{2T}{(2\pi)^3 Nm} \sum_{\omega} \int \vec{p}'_\alpha \, \mathrm{d}^3\vec{p}' \, \vec{\Pi}^{(1)}_\beta(p'_+, p'_-). \tag{2}$$

$p'_\pm = p' \pm \frac{q}{2}$, and $\vec{\Pi}^{(1)}(p_+, p_-)$ is the Fourier component

$$\begin{aligned}\vec{\Pi}^{(1)}(x-y, y-x') = {} & \frac{T^2}{(2\pi)^6} \sum_{\omega_+,\omega_-} \int\int \vec{\Pi}^{(1)}(p_+, p_-) \mathrm{e}^{\mathrm{i}\vec{p}_+(x-y) - \mathrm{i}\omega_+(\tau_x - \tau_y)} \\ & \times \mathrm{e}^{\mathrm{i}\vec{p}_-(y-x') - \mathrm{i}\omega_-(\tau_y - \tau'_x)} \, \mathrm{d}^3\vec{p}_+ \, \mathrm{d}^3\vec{p}_- \\ \vec{\Pi}^{(1)}(x-y, y-x') = {} & \frac{-\mathrm{i}}{2} (\vec{\nabla}_y - \vec{\nabla}_{y'})_{y'\to y} [\overline{G(x, y')G(y, x')} - \overline{F^+(y', x')F(x, y)}]\end{aligned} \tag{3}$$

where G and F are the Green functions in the superconducting state, and the wide bars denote averages over the impurity positions.

For calculating the response tensor $Q_{\alpha\beta}$, we may define the function $B^{(i)}(\omega')$ as

$$B^{(i)}(\omega') = \frac{n_i}{(2\pi)^3} \int |u(\vec{p} - \vec{p}')|^2 \vec{\Pi}^{(i)}(\vec{p}'_+, \vec{p}'_-) \, \mathrm{d}^3\vec{p}' \tag{4}$$

where $u(\vec{p} - \vec{p}')$ is the impurity potential [2], and n_i is the density of impurities.

To determine $\Pi^{(1)}(p_+, p_-)$, we have to know three further quantities that differ in the diagram from $\Pi^{(1)}(p_+, p_-)$ by having different directions of the arrows on the electron line. Each of these quantities corresponds to a special combination of G and F:

$$\begin{aligned}\Pi^{(2)}(x-y, y-x) &= -\frac{\mathrm{i}}{2} (\nabla_y - \nabla_{y'})_{y\to y'} [\overline{F^+(x, y')G(y, x')} + \overline{G(y, x)F^+(y', x')}] \\ \Pi^{(3)}(x-y, y-x') &= -\frac{\mathrm{i}}{2} (\nabla_y - \nabla_{y'})_{y'\to y} [\overline{G(y, x)G(x', y')} - \overline{F^+(x, y')F(y, x')}] \\ \Pi^{(4)}(x-y, y-x') &= -\frac{\mathrm{i}}{2} (\nabla_y - \nabla_{y'})_{y'\to y} [\overline{G(x, y')F(y, x')} + \overline{F(x, y)G(x', y')}].\end{aligned} \tag{5}$$

With the help of Feynman diagrams, the equation for $\vec{\Pi}^{(1)}$ is given by [2]

$$\vec{\Pi}^{(1)}(p_+, p_-) = \vec{P}\{G(p_+)G(p_-) + F(p_+)F^+(p_-)\} + G(p_+)G(p_-)B^{(1)}(\omega) - F^+(p_+)G(p_-)B^{(2)}(\omega) - F(p_+)F^+(p_-)B^{(3)}(\omega) - G(p_+)F(p_-)B^{(4)}(\omega). \tag{6}$$

Substituting equation (5) into (4) leads to a system of equations for the $B^{(i)}(\omega)$. It can be solved in general form only in the case of spherically symmetric scattering. The criterion as to whether the superconductor is local or not is based on the concentration of impurities. At small concentrations, the properties of a superconductor are close to those of a pure metal. We shall assume that the impurity concentration is such that the superconductor has not become London type. In this situation, for spherically symmetric scattering, by using

$$G(p) = -\frac{\mathrm{i}\omega - \bar{G}_\omega + \varepsilon}{(\mathrm{i}\omega - \bar{G}_\omega)^2 + \varepsilon^2 + (\Delta_p + \bar{F}^+_\omega)^2}$$
$$F^+(p) = \frac{\Delta_p + \bar{F}^+_\omega}{(\mathrm{i}\omega - \bar{G}_\omega)^2 + \varepsilon^2 + (\Delta_p + \bar{F}^+_\omega)^2} \tag{7}$$

where

$$\bar{G}_\omega = \frac{n_i}{(2\pi)^3}\int |u(\vec{p} - \vec{p}')|^2 G(\vec{p}')\,\mathrm{d}^3 p'$$
$$\bar{F}^+_\omega = \frac{n_i}{(2\pi)^3}\int |u(\vec{p} - \vec{p}')|^2 F^+(\vec{p}')\,\mathrm{d}^3 \vec{p}'$$

we may write

$$B^{(1)}(\omega) = -B^{(3)}(\omega) = \frac{\Delta_p^2}{2\tau[\omega^2 + \Delta_p^2 + (\frac{1}{2}\vec{q}\cdot\vec{v})^2]\left(\sqrt{\omega^2 + \Delta_p^2} + \frac{1}{2\tau}\right)}$$
$$B^{(2)}(\omega) = B^{(4)}(\omega) = \frac{\mathrm{i}\Delta_p\omega}{2\tau[\omega^2 + \Delta_p^2 + (\frac{1}{2}\vec{q}\cdot\vec{v})^2]\left(\sqrt{\omega^2 + \Delta_p^2} + \frac{1}{2\tau}\right)} \tag{8}$$

where the gap parameter in d-wave superconductors is given by $\Delta_p = \Delta_0 \cos 2\varphi$ (Δ_0 is the maximum gap parameter, and φ is the angular deviation of $\hat{p}$ from the given node direction in the basal plane), and τ is the relaxation time in the superconducting state.

If the electromagnetic response tensor $Q_{\alpha\beta}$ is diagonal, it is simply related to the eigenvalues of the penetration depth tensor $\frac{4\pi}{c}Q_{\alpha\alpha}(q) = \lambda_\alpha^{-2}$, where λ_α is the penetration depth for current flow in the direction α. By using equations (8) and (4) in equation (2), we obtain the kernel $Q(q)$ as

$$Q(q) = \Bigg\{1 + \tfrac{3}{4}T\sum_\omega \int \mathrm{d}\varepsilon \Bigg[((\varepsilon_+ + \mathrm{i}\omega\gamma_\omega)(\varepsilon_- + \mathrm{i}\omega\gamma_\omega) + \Delta_p^2\omega^2) \times \left(1 + \frac{\Delta_p^2}{2\tau(\omega^2 + \Delta_p^2)^{3/2}\gamma_\omega} + \frac{2\Delta_p^2\omega^2\gamma_\omega^2}{2\tau(\omega^2 + \Delta_p^2)^{3/2}\gamma_\omega}\right)\Bigg] \times \frac{1}{[\varepsilon_+^2 + (\omega^2 + \Delta_p^2)\gamma_\omega^2][\varepsilon_-^2 + (\omega^2 + \Delta_p^2)\gamma_\omega^2]}\Bigg\} \tag{9}$$

where $\gamma_\omega = 1 + (2\tau\sqrt{\omega^2 + \Delta_p^2})^{-1}$ and $\varepsilon_\pm = \varepsilon \pm \frac{1}{2}\vec{q}\cdot\vec{v}$.

Here we encounter a formally divergent integral. After regrouping the terms in the curved brackets, we cancel the divergent terms, and after some algebra we obtain

$$Q(\tilde{q}, T) = 2\pi T\sum_\omega \left\langle \widehat{p}_{11}^{\,2} \frac{\Delta_p^2}{(\omega^2 + \Delta_p^2 + \alpha^2)\left(\sqrt{\omega^2 + \Delta_p^2} + \frac{1}{2\tau}\right)}\right\rangle \tag{10}$$

where $\widehat{p}_{11}$ is the projection of $\hat{p}$ on the boundary, $\alpha = (\frac{qv_F}{2})\hat{p}\cdot\hat{q}$ ($\hat{q}$ is the unit vector perpendicular to the boundary, and it gives the direction in which the penetration of the magnetic field takes place), $\tilde{q} = \lambda_0 q$, and

$$\langle\cdots\rangle = \frac{1}{\pi R_f^2}\int_0^{R_f}\int_0^{2\pi}\sin\varphi\,\mathrm{d}\varphi\,r\,\mathrm{d}r.$$

We consider two different orientations of the magnetic field. First, the static magnetic field is applied along the c-axis with a superconducting plane perpendicular to the c-axis. For this particular geometry, both the vector potential $\vec{A}$ and the screening supercurrent density $\vec{J}$ are parallel to the a-axis. In this situation $\alpha \neq 0$, and equation (10) shows the effects of nonlocality and impurity on the both penetration depths $\Delta\lambda_{ab}(T)$ and $\Delta\lambda_c(T)$. Second, in a different geometry where the boundary is parallel to the a–b plane ($\vec{H}$ parallel to the boundary), the direction of penetration $\hat{q}$ would be along the c-axis, i.e., perpendicular to $\hat{p}$, yielding $\alpha = 0$; hence equation (10) shows the effects of impurity on both penetration depths.

Now we calculate the correction of $Q(\tilde{q}, T)$ in the limit of low temperatures, in the presence of nonlocality and impurity. For this purpose $Q(\tilde{q}, T)$ may be written in terms of $Q(\tilde{q}, 0)$ as

$$Q(\tilde{q}, T) = Q(\tilde{q}, 0) + \delta Q(\tilde{q}, T) \tag{11}$$

where

$$\delta Q(\tilde{q}, T) = 2\left\{\delta Q(0, T)\left[1 + \frac{2}{\delta Q(0, T)}\int_0^\infty \mathrm{d}\omega\, f(\omega) \times\left\langle 2\widehat{p}_{11}^2\Delta_p^2\frac{1}{(\omega^2-\Delta_p^2)^{3/2}}\frac{\alpha^2}{\omega^2-\Delta_p^2-\alpha^2}\right\rangle\right]\right\} \tag{12}$$

with

$$\delta Q(0, T) = 2\int_0^\infty f(\omega)\,\mathrm{d}\omega\left\langle 2\widehat{p}_{11}^2\Delta_p^2\frac{1}{(\omega^2-\Delta_p^2)^{3/2}}\frac{1}{\left(1+\frac{1}{2\tau\sqrt{\omega^2-\Delta_p^2}}\right)}\right\rangle. \tag{13}$$

Equation (12) can be written as

$$\delta Q(\tilde{q}, T) = \delta Q(0, T)\, F\left(\frac{\tilde{q}}{t}\right) \tag{14}$$

where

$$F\left(\frac{\tilde{q}}{t}\right) = 1 + \frac{2}{\delta Q(0, T)}\int_0^\infty f(\omega)\,\mathrm{d}\omega\left\langle 2\widehat{p}_{11}^2\Delta_p^2\frac{1}{(\omega^2-\Delta_p^2)^{3/2}}\frac{\alpha^2}{\omega^2-\Delta_p^2-\alpha^2}\right\rangle, \tag{15}$$

and $t = T/T^*$.

Close to the nodes, $\Delta_p \approx 2\Delta_0\varphi$, and we have

$$\delta Q(0, T) = -2T\tau\ln 2. \tag{16}$$

It is noted that nonlocality parameter, α, appeared only in the function $F(\frac{\tilde{q}}{t})$, which is the same as the result of Kosztin and Leggett [7], whereas the relaxation time τ appears only in the function $\delta Q(0, T)$. It is noted that $\delta Q(0, T)$ is the same as that of Kosztin and Leggett when $\tau \to \infty$. Here $F(z)$ is a universal function which can be approximated by [7],

$$F(z) \approx \begin{cases} 1 - c_1 z & \text{for } z < 2 \\ \dfrac{c_2}{z^2} & \text{for } z > 2 \end{cases}$$

where $c_1 = 0.37$, $c_2 = 1.05$, $z = \frac{\tilde{q}}{t}(T^* = \frac{\zeta_0}{\lambda_0}\Delta_0)$, and ζ_0 is the coherence length.

For specular and diffuse boundary scattering respectively we have [7]

$$\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{2}{\pi}\int_0^\infty \mathrm{d}\tilde{q}\,\frac{[-\delta Q(\tilde{q},T)]}{(\tilde{q}^2+1)^2} \tag{17}$$

$$\frac{\Delta\lambda_{diff}(T)}{\lambda_0} = \frac{1}{\pi}\int_0^\infty \mathrm{d}\tilde{q}\,\frac{[-\delta Q(\tilde{q},T)]}{\tilde{q}^2-1}. \tag{18}$$

Because in the range of temperatures $T \ll T^*$, $\tilde{q} \ll 1$, $\tilde{q}^2$ in the denominators of the above equations can be neglected. However, $t \ll 1$, and the upper limit of the integral is changed to t [9]; hence finally we may write

$$\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{4\tau\ln 2T}{\pi}\int_0^t \mathrm{d}\tilde{q}\,\frac{1-0.37\frac{\tilde{q}}{t}+1.052\frac{t^2}{\tilde{q}^2}}{(\tilde{q}^2+1)^2}.$$

By integrating we may obtain

$$\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{7.52\tau\ln 2}{\pi}\frac{T^2}{T^*}. \tag{19}$$

The relaxation time, τ, for d-wave superconductors in the low-temperature limit, is given by [8]

$$\tau \approx \tau_n\cos^2\delta_N|\cot g\delta_N|\left|\frac{\Delta_0}{E}\right|\frac{1}{|\ln(\frac{2\Delta_0}{E})|} \tag{20}$$

where δ_N is the phase shift; τ_n is the relaxation time in the normal state, and in the Born approximation is given by [8]

$$\tau_N^{-1} = \frac{n_i m p_F}{(2\pi)^2}\int |u(\theta)|^2\,\mathrm{d}\Omega \tag{21}$$

where p_F is the Fermi wavevector.

If we depart from the Born approximation, we have to take into account the diagrams containing several crosses per impurity atom. It can be shown that the resulting change is simply the replacement of the Born amplitude $u(\theta)$ by the total scattering amplitude t_N:

$$\tau_N^{-1} = 2\pi n_i N(0)|t_N|^2 \tag{22}$$

where $N(0) = \frac{mp_F}{2\pi^2}$ is the density of quasi-particle states at the Fermi surface for a single spin; t_N is the total scattering amplitude, and is given by [8]

$$|t_N| = \frac{\sin\delta_n}{\pi N(0)}. \tag{23}$$

For $E \ll \Delta_0$, one may finally write

$$\tau \approx \frac{N(0)}{2n_i}(\cot g^3\delta_N)\frac{\Delta_0}{T}. \tag{24}$$

By substituting equation (24) into equation (19), we obtain

$$\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{3.8\ln 2\Delta_0}{\pi}\frac{N(0)}{n_i}\cot g^3\delta_N\frac{T}{T^*}. \tag{25}$$

A similar calculation in the case of a diffuse boundary yields

$$\frac{\Delta\lambda_{diff}(T)}{\lambda_0} \approx \frac{1}{2}\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{1.9\ln 2\Delta_0}{\pi}\frac{N(0)}{n_i}\cot g^3\delta_N\frac{T}{T^*}. \tag{26}$$

This behaviour of the relaxation time gives rise to transport coefficients that are in qualitative disagreement with experiments [4]. Schmitt *et al* [5], have performed calculations for the transport properties of anisotropic superconductors assuming the electron impurity scattering to be close to resonant, and therefore not treatable in the Born approximation. They find at low energies $\tau \propto \tau_N$ and nearly equal to τ_N in the energy region above the pair-breaking regime relevant to most of the existing experiments. By using this result, for the penetration depth we may write

$$\frac{\Delta\lambda_{spec}(T)}{\lambda_0} = \frac{2\Delta\lambda_{diff}(T)}{\lambda_0} \approx \frac{7.5\tau_N \ln 2}{\pi}\frac{T^2}{T^*} \tag{27}$$

which is in agreement with the experimental results [6, 10, 11].

In the limit of $\tau \to \infty$ (pure superconductors), equation (10) shows only the effects of nonlocality on the temperature dependence of the penetration depth, and this is consistent with the results of Kosztin and Leggett [7].

In the limit of $\alpha \to 0$ (local limit), equation (10) shows only the effects of impurity on the penetration depth. In this case we have

$$Q(\tilde{q}, T) = 2\pi T \sum_{\omega} \left\langle \widehat{p}_{11}^{2} \frac{1}{(\omega^2 + \Delta_p^2)} \frac{\Delta_p^2}{\left(\sqrt{\omega^2 + \Delta_p^2} + \frac{1}{2\tau}\right)} \right\rangle. \tag{28}$$

When $\frac{1}{\tau} \to 0$, the formula transforms to the usual London-type expression:

$$\lambda(T) = \left(\frac{m}{4\pi N_s e^2}\right)^{1/2}, \tag{29}$$

where N_s is the number of supercomputing electrons.

3. Discussion and concluding remarks

We calculated $Q(\tilde{q}, T)$ in the presence of nonlocality and impurity. We conclude that in the limit of $\tau \to \infty$ our results are the same as those of Kosztin and Leggett. In the limit of $\alpha \to 0$ (local limit) the effects of impurity only appeared in the temperature dependence of $\Delta\lambda(T)$. In the presence of both nonlocality and impurity the nonlocal effects are in fact masked by impurities at low temperatures.

We showed that if electron–impurity scattering is treated in the Born approximation, $\Delta\lambda(T)$ varies linearly with temperature which is in qualitative disagreement with experiments. On the other hand, if instead of using the Born approximation we assume that multiple interactions of a quasi-particle with impurities are important, which corresponds to the phase shift in the normal state being close to $\frac{\pi}{2}$, the qualitative temperature dependence of $\Delta\lambda(T)$ goes to T^2, which is in agreement with experimental results. If resonant scattering is responsible for the quadratic temperature dependence of λ in an impure sample, in the presence of nonlocality and of impurity effects, nonlocal effects are in fact completely masked by impurities.

References

[1] Fiory A T *et al* 1988 *Phys. Rev. Lett.* **61** 1919

[2] Abrikosov A A, Gorkov L P and Dzyaloshinski I E 1963 *Methods of Quantum Field Theory in Statistical Physics* (Englewood Cliffs, NJ: Prentice-Hall)

[3] Hirschfeld J P and Goldenfeld N 1993 *Phys. Rev.* B **48** 4219

[4] Bishop D J, Varma C M, Batlogg B, Bucher E, Fisk Z and Smith J L 1984 *Phys. Rev. Lett.* **53** 1009

[5] Schmitt S, Miyake K and Varma C M 1986 *Phys. Rev. Lett.* **57** 2575

[6] Bonn D A *et al* 1994 *Physica* B **50** 4051

[7] Kosztin I and Leggett A J 1997 *Phys. Rev. Lett.* **79** 135

[8] Arfi B, Bahloli H and Pethick C J 1989 *Phys. Rev.* B **39** 8959

[9] Shahzamanian M A 1989 *J. Phys.: Condens. Matter* **1** 1965

[10] Djordjevic S, Farber E, Deutscher G, Bontemps N, Durand O and Contour J P 2002 *Eur. Phys. J.* B **25** 407

[11] Prozoruv R and Giannetta R W 2000 *Phys. Rev. Lett.* **85** 3700